LITERATURE CITED

1. V. K. Shchukin, Heat Transfer and Hydrodynamics of Internal Flows in Mass-Force Fields [in Russian], Mashinostroenie, Moscow (1970).
2. S. Abell and J. L. Hudson, Int. J. Heat Mass Transfer, 18, No. 12 (1975).
3. A. D. Id, in: Developments in Heat Transfer (edited by W. M. Rohsenow), Massachusetts Institute of Technology (1964).
4. I. M. Pchelkin, in: Convective and Radiant Heat Transfer [in Russian], AN SSSR, Moscow (1960).
5. V. K. Shchukin and V. V. Olimpiev, Inzh.-Fiz. Zh., 30, No. 4 (1976).

MODELING TURBULENT FLOW AT A PERMEABLE PLATE WITH STRONG INJECTION

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It is shown that available experimental data on the thickness of the turbulent-boundary layer and the filling of the velocity profile under strong injection are satisfactorily generalized using the parameter $\rho_w v^2_w/\rho_0 u_0^2$.

The boundary layer at a permeable surface has been investigated in a large number of works but as yet no satisfactory answers have been given to a series of important questions arising from the study of such flows. In particular, there is no agreement as to the existence and form of a universal parameter determining the boundary-layer thickness at a permeable plate under strong injection.

Strong injection is taken to be such that the velocity profile degenerates to a straight line and the concentration of injected material at the permeable wall approaches 100%.

Most researchers believe that the thickness of the turbulent-boundary layer at a permeable plate is uniquely determined by the parameter $(\rho_w v_w/\rho_0 u_0)\cdot \mathrm{Re}^{0.2}$, regardless of the strength of injection. This view is based on experimental results obtained with intermixing gas flows either of the same density or else of only slightly different densities [1-5].

Boundary layers with intermixing gas flows of significantly different densities were first systematically investigated in [2, 3]. It was argued that the generally used injection parameter should be modified by the introduction of a factor with a variable index so as to take into account the density ratio. However, only results obtained for the thermal state of a permeable plate were generalized using this parameter.

Systematic data were given in [6] on the thickness of the turbulent boundary layer at a permeable plate when the intermixing flows are of significantly different densities. It was established that the boundary-layer thickness and the filling of the velocity profile depend on the parameter $(\rho_w v_w/\rho_0 u_0)(\rho_w/\rho_0)^k \cdot \mathrm{Re}^{0.2}$, where $k = -0.25^{\pm 0.05}$ for $1 \le \rho_w/\rho_0 \le 3$ and $k = -0.5^{\pm 0.1}$ for $0.07 \le \rho_w/\rho_0 \le 1$, regardless of the injection strength. It was shown that the same parameter also determines the separation of the boundary layer from the wall.

In [7, 8], an interferometer was used to determine the conditions for the separation of the turbulent-boundary layer at a porous plate with the injection of heterogeneous gas. As a result it was possible to confirm that the separation parameter $(\rho_w v_w/\rho_0 u_0)\cdot \mathrm{Re}^{0.2}$ depends strongly on the density ratio of the intermixing flows.

Thus it has been shown that under strong injection the parameter $(\rho_w v_w/\rho_0 u_0)\cdot \mathrm{Re}^{0.2}$ does not uniquely determine the flow in a turbulent-boundary layer at a permeable plate. Note, however, that empirical parameters with a variable index in the factor $(\rho_w/\rho_0)^k$, such as are introduced in [3, 6], do not have a clear physical meaning, since they derive neither from boundary-layer theory nor from jet theory. Therefore, neither of them can be regarded as a general parameter.

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Consider the hypothetical flow forming from the mixing of gas flows issuing from two permeable plates inclined at an angle of $\pi/2$ to one another and having a common point.

The important feature of this flow is the presence of a mixing zone, which occupies an angle much smaller than $\pi/2$.

Depending on the parameters of the intermixing flows this region may occupy any position within the region under consideration.

By analogy with the propagation of a transverse jet in a carrier flow, assume that, in the region where the walls have no effect, the position of the mixing zone is determined by the parameter $\rho_w v_w^2/\rho_0 u_0^2$.

Hence it is evident that at points of separation of the boundary layer from the wall, where the effect of the wall vanishes, the position of the mixing zone will again be determined by this parameter and will not also depend on the density ratio, in view of the symmetry of the problem with respect to each of the plates.

It has been shown experimentally [8] that this parameter also determines the separation of the boundary layer from the wall.

It was shown in [6] that for large subcritical injection the shape parameter and the separation parameter are of the same form. Evidently, there is a region close to critical injection where the position of the mixing-zone boundary is determined by the velocity ratio and does not depend on Re.

In the absence of Re from the determining parameter [8], it may be assumed that the form of the flow at this level of injection is determined mainly by forces due to the pressures of the intermixing flows, despite the increase (noted in [8]) in friction in the mixing zone as injection increases.

On the basis of the fundamental principle of similarity theory, an attempt will now be made to derive a parameter determining the boundary-layer thickness at a permeable plate under strong injection and to obtain an approximate estimate of this thickness. Assume that the frictional forces under strong injection may be neglected; i.e., the flow is mainly determined by forces due to the pressure.

The resultant force acting on an arbitrary elementary volume of liquid within the boundary layer may be resolved into two components $F_x = \rho\,(Du/Dt)$ and $F_y = \rho\,(Dv/Dt)$. Since the flow is mainly determined by forces due to the pressure, the similarity condition for strong injection is evidently simply the equality $F_x/F_y = \text{const}$. For the case of steady flow

$$F_x = \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) \quad \text{and} \quad F_y = \rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right).$$

It is of interest to estimate the order of magnitude of these components.

As shown in [9], the right-hand sides of these expressions consist of terms of the same order, i.e., $u\,(\partial u/\partial x) \sim v\,(\partial u/\partial y)$ and $u\,(\partial v/\partial x) \sim v\,(\partial v/\partial y)$. It was confirmed experimentally in [4] that this result applies to a boundary layer with injection. Therefore, the order of magnitude of F_x and F_y may be estimated from one of the terms, i.e., $F_x \sim \rho u(\partial u/\partial x)$ and $F_y \sim \rho v\,(\partial v/\partial y)$. For a plate of length x, $\partial u/\partial x$ is proportional to u_0/x.

Taking the carrier-gas flow velocity u_0 as the horizontal component of the characteristic velocity, and the density of this flow ρ_0 as the characteristic density, it is found that the horizontal component of the resultant force acting on an elementary volume of liquid is $\rho_0 u_0^2/x$ [9].

Now consider the vertical component F_y.

The vertical component of the characteristic velocity is taken to be the injected-gas velocity v_w.

The injected-gas density ρ_w is taken as the characteristic density. The characteristic linear dimension may evidently be taken to be $\delta - \delta_0$, where δ_0 is the boundary-layer thickness in the absence of injection.

Thus, the vertical component of the force acting on an elementary volume of liquid is of the order of $\rho_w v_w^2/(\delta - \delta_0)$. Then the similarity condition for the flow at a permeable plate may be written as follows: $(\rho_0 u_0^2/\rho_w v_w^2)\cdot(\delta-\delta_0)/x = k$ or $\delta - \delta_0 = kx(\rho_w v_w^2/\rho_0 u_0^2)$. In this relation there remains the indeterminate factor k on the right-hand side.

By analogy with the results for the boundary-layer thickness at an impermeable plate [9], it may be assumed that for turbulent flow only the structure of the formula obtained is correct, since the parameter included in the formula must be raised to some power m. The value of m may evidently be determined from any other considerations or from experiment.

TABLE 1. Experimental Conditions for Literature Data

Surface form	Lit. source	Notation	$\frac{\rho_w}{\rho_0}$	Re_0	x, mm	T_0, °K	U_0 m/sec	Injected gas
Perforations	[6]	1	0,138	$2{,}6 \cdot 10^5$	90	280	41,5	Helium
		2	0,069	$3{,}3 \cdot 10^5$	95	281	49	Hydrogen
		3	1	$2{,}5 \cdot 10^5$	90	290	42	Air
		4	1	$5{,}1 \cdot 10^5$	90	292	87	"
		5	1	$9{,}8 \cdot 10^5$	90	291	164	"
		6	1,77	$1{,}2 \cdot 10^5$	90	513	53	"
		7	1,68	$2{,}7 \cdot 10^5$	90	487	113	"
		8	1,67	$4{,}9 \cdot 10^5$	90	485	197	"
		9	2,37	$4{,}9 \cdot 10^5$	90	290		Argon
Pores	[1]	10	1	$6 \cdot 10^5$	1060	290	7,9	Air
	[4]	11	1	$2 \cdot 10^5$	2500	290	14	"
	[4]	12	1	$4 \cdot 10^5$	450	290	14	"

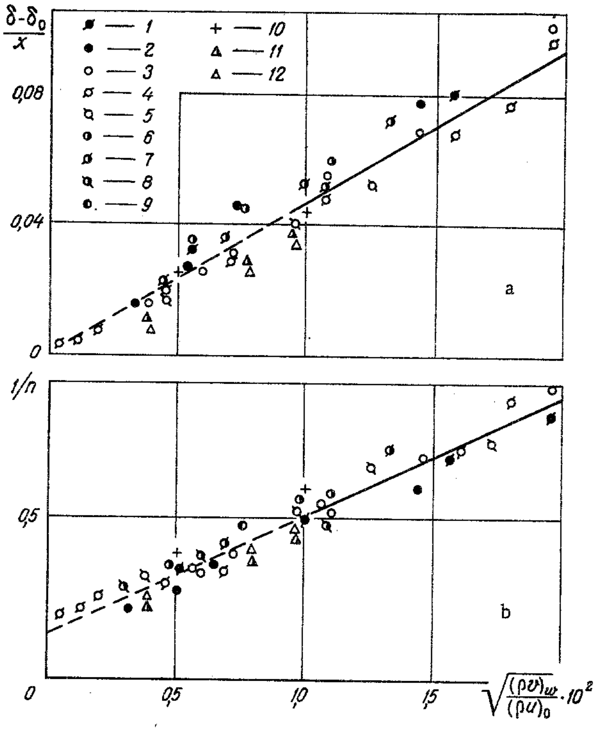

Fig. 1. Dependence of relative boundary-layer thickness (a) and shape of velocity profile (b) on injection parameter. The notation is given in Table 1.

Analysis of the data of [1, 4, 6] shows that for fixed $\rho_0 u_0^2$ the measured values of the boundary-layer thickness may be generalized with good accuracy in the coordinates $\delta - \delta_0$, $(\rho_w v_w^2)^{0.5}$. Hence it follows that the appropriate value of m is 0.5.

The formula for the calculation of the boundary-layer thickness at a permeable plate then takes the form $\delta - \delta_0 = kx(\rho_w v_w^2/\rho_0 u_0^2)^{0.5}$.

The next step is to attempt a generalization of the available literature data on the boundary-layer thickness and the filling of the velocity profile using the parameter $\rho_w v_w^2/\rho_0 v_0^2$.

Note that, despite the large number of investigations of the boundary layer at permeable plates, the number containing reliable experimental data remains extremely small. Most of the results are of limited significance, because of the previous history of the gas flow arriving at the plate and the presence of a large pressure gradient along the plate.

The results of classical experiments [1, 4] on intermixing flows of the same density, which are known to be free of these limitations, and the results of the unique experiments of [6] on intermixing flows of significantly different densities are considered below; the corresponding experimental conditions are given in Table 1.

These results were obtained by approximating the experimental velocity profile by a power law of the form $u/u_0 = (y/\delta)^{1/n}$. In Fig. 1, dependences of $(\delta - \delta_0)/x$ (a) and $1/n$ (b) on the parameter $(\rho_w v_w^2/\rho_0 v_0^2)^{0.5}$, obtained in [1, 4, 6], are shown.

It is evident from Fig. 1 that the experimental data for strong injection may be generalized, with satisfactory accuracy, using the parameter $\rho_w v_w^2/\rho_0 v_0^2$. As is evident, the approximating straight line may be satisfactorily extrapolated to the weak-injection region (dashed lines).

In [2, 5-8], it was established that strong injection begins when the injection parameter is close to the value $(\rho_w v_w^2/\rho_0 u_0^2)^{0.5} = 0.02$. The fact that in the region $0.01 < (\rho_w v_w^2/\rho_0 u_0^2)^{0.5} < 0.02$ close to the beginning of strong injection the experimental data may be generalized using a parameter obtained in a consideration of the jet problem confirms the assumption that viscous forces have no significant effect on the form of the flow in such boundary layers.

The scatter of the experimental data in the weak-injection region $0.001 < (\rho_w v_w^2/\rho_0 u_0^2)^{0.5} < 0.01$ prevents any definite conclusion as to the form of the flow. It is only possible to note that on the basis of these data there is no reason to expect a dominant effect of viscous forces in this region.

NOTATION

x, y, longitudinal and transverse coordinates; u, v, longitudinal and transverse velocity components; ρ, density; δ, boundary-layer thickness; δ_0, boundary-layer thickness in the absence of injection; Re, Reynolds number based on plate length and parameters at boundary-layer edge. Indices: 0, external edge of boundary layer; w, wall.

LITERATURE CITED

1. H. S. Mickley, R. E. Ross, A. L. Squier, and W. E. Stewart, Nat. Advis. Comm. Aeronaut. Tech. Note No. 3208 (1954).
2. V. P. Mugalev, Izv. Vyssh. Uchebn. Zaved., Aviats. Tekh., No. 3 (1959).
3. V. P. Mugalev, Heat and Mass Transfer [in Russian], Vol. 1, Énergiya, Moscow (1968).
4. R. L. Simpson, J. Fluid Mech., 42, Part 4 (1970).
5. B. P. Mironov, Author's Abstract of Doctoral Dissertation, Novosibirsk (1970).
6. V. V. Glazkov, M. D. Guseva, and B. A. Zhestkov, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 4 (1972).
7. V. M. Eroshenko, A. L. Ermakov, A. A. Klimov, and V. P. Motulevich, Inzh.-Fiz. Zh., No. 1 (1972).
8. V. M. Eroshenko, Author's Abstract of Doctoral Dissertation, Moscow (1975).
9. H. Schlichting, Boundary Layer Theory, McGraw-Hill, New York (1967).